

Article

Mobility Data Warehouses

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Abstract: The interest in mobility data analysis has grown dramatically with the wide availability of devices that track the position of moving objects. Mobility analysis can be applied, for example, to analyze traffic flows. To support mobility analysis, trajectory data warehousing techniques can be used. Trajectory data warehouses typically include, as measures, segments of trajectories, linked to spatial and non-spatial contextual dimensions. This paper goes beyond this concept, by including, as measures, the trajectories of moving objects at any point in time. In this way, online analytical processing (OLAP) queries, typically including aggregation, can be combined with moving object queries, to express queries like “List the total number of trucks running at less than 2 km from each other more than 50% of its route in the province of Antwerp” in a concise and elegant way. Existing proposals for trajectory data warehouses do not support queries like this, since they are based on either the segmentation of the trajectories, or a pre-aggregation of measures. The solution presented here is implemented using MobilityDB, a moving object database that extends the PostgreSQL database with temporal data types, allowing seamless integration with relational spatial and non-spatial data. This integration leads to the concept of mobility data warehouses. This paper discusses modeling and querying mobility data warehouses, providing a comprehensive collection of queries implemented using PostgreSQL and PostGIS as database backend, extended with the libraries provided by MobilityDB.

Keywords: mobility; data warehouses; spatiotemporal OLAP; mobility analytics

1. Introduction

Moving objects [1] (MOs) are objects (e.g., cars, trucks, pedestrians) whose spatial features change continuously in time. The data produced by MOs (e.g., by using attached devices like GPSs and smartphones) can be analyzed in many ways, for example, to discover mobility patterns [2]. This is called mobility data analysis, a technique that is currently used in many related fields, like traffic management, transportation, car pooling applications, smart cities, etc.

The increasing availability of MO data has triggered the interest in mobility data analysis, which has been growing steadily year after year. Moving object data generally come in the form of long sequences of spatiotemporal coordinates $\langle x, y, t \rangle$. To facilitate analysis, these sequences are split into smaller portions of movement, called trajectories. Trajectories can be represented in two possible ways. A continuous trajectory represents the movement track of an object by means of a sequence of spatiotemporal points occurring within a certain interval, together with interpolation functions that allow computing the (approximate) position of the object at any instant. A discrete trajectory contains only a sequence of spatiotemporal points but no interpolation function can be defined.

Moving object databases are databases that store the positions of MOs at any point in time. Although these databases are appropriate for querying, for example, current movement, by means of

queries like “which taxis are within ten minutes from Brussels Central Station”, they do not support, per se, complex analytical queries such as “For each month, list the total number of buses running at a speed higher than 60 km per hour for more than 40% of their total route.” For this, integration with, for example, data warehousing technologies, is needed, yielding the notion of mobility data warehouses. These are data warehouses that contain MO data that can be analyzed in conjunction with other kinds of data (e.g., spatial data), for instance, a road network, altitude data, and the kind.

To represent MOs, the definition of appropriate data types is needed. The notion of temporal type refers to a collection of data types that capture the evolution over time of base types and spatial types. For instance, temporal integers may be used to represent the evolution in the number of employees in a department. Analogously, a temporal point may represent the evolution in time of the position of a vehicle, reported by a GPS device, which would yield a temporal geometry of type point. Over these kind of data types, MO databases can be implemented. Thus, MobilityDB was developed (the demo of this database can be accessed at <http://demo.mobilitydb.eu/>, where also the manual with a full description of the data types is available). MobilityDB is a MO database (MOD), that builds on PostGIS (the spatial extension of PostgreSQL), that extends the type system of PostgreSQL and PostGIS with abstract data types (ADTs), in order to represent MO data. These ADTs are based on the notion of temporal types and their associated operations.

1.1. Mobility Data Warehouses

Conventional databases are used to support the day-to-day operations in an organization. The operations over these databases are usually denoted as online transactional processing (OLTP). To support data analysis, the notion of data warehousing was developed. A data warehouse (DW) collects large amounts of data from various data sources and reduces them to a form that can be used to analyze the behavior of an organization. DWs are based on the multidimensional model, which represents data as facts that can be analyzed along a collection of dimensions, composed of levels conforming aggregation hierarchies. The multidimensional model builds on the data cube abstraction, where the axes of the cubes are the dimensions, and the cells contain measure values. Typically, DWs are exploited by means of the online analytical processing (OLAP) technique, which consists of a collection of operations that manipulate the data cube. The most popular OLAP operations are roll-up, which aggregates measure data along a dimension up to a certain aggregation level; drill-down, which is the inverse of the former; slice, which drops a dimension from the cube; and dice, which selects a sub-cube that satisfies a boolean condition.

Spatial data can represent geographic objects (e.g., mountains, cities), geographic phenomena (e.g., temperature, precipitation), etc. Similarly to conventional databases, spatial databases are typically used for operational applications in many different domains, rather than to support data analysis tasks. Spatial DWs (SDWs), on the other hand, combine spatial databases features and DW technologies, to provide more sophisticated data analysis, visualization, and manipulation capabilities. Thus, a SDW is a DW that is capable of representing, storing, and manipulating spatial data. Usually, to represent the extent of a spatial object, spatial data types are used. A SDW can thus take advantage of the operations associated with spatial data types, to allow queries like “give me the total number of theaters within one kilometer from my current position”.

Most DWs (either spatial or not) assume that only facts evolve in time. However, dimension data, like for instance, the category of a product, may also change across time. The most popular approach for tackling this issue is based on the notion of slowly changing dimensions [3]. An alternative approach for this issue is based on the concept of temporal databases. Built-in temporal semantics allow these kinds of databases to manage time-varying information. The combination of spatial and temporal databases and DWs leads to the notion of spatiotemporal DWs. Since MO are in essence spatiotemporal, adding MO data to a DW leads to the notion of mobility DWs. This is, in a nutshell, the problem addressed by this paper: modeling and querying mobility DWs by means of adding temporal types to spatial DWs.

1.2. Contributions and Motivation

The authors of this paper had previously defined the notion of spatiotemporal queries as queries that can be expressed by Klug's relational algebra with aggregation, extended with spatial and moving types [4]. Based on this, the authors defined *spatiotemporal DWs* as DWs that support spatiotemporal queries. The classification in the referred work helps to characterize the different kinds of DW systems in terms of their expressiveness, and it is used in the discussion below.

Many efforts have been published under the concept of trajectory data warehouses [5–7]. These works basically propose storing, in the DW fact tables, aggregate measures (like the number of cars in a given latitude-longitude range), segments of trajectories, or spatiotemporal points, probably together with semantic information. It is therefore clear that these approaches do not qualify as trajectory or, equivalently, spatiotemporal DWs (according to the classification in [4]), since they do not support spatiotemporal queries, given that they do not include moving data types. For example, storing aggregate measures [6], does not allow queries like “total distance covered by all cars in Brussels”. Instead, they allow expressing queries like “average number of cars in downtown Brussels on Sunday mornings”. These approaches conform, in some sense, traditional spatial DWs. The other proposals (although implementations are not yet reported), aim at querying semantic trajectories, for example “total number of persons going from restaurants to theaters”. Even in the case where spatiotemporal points are stored, these represent the raw trajectories rather than moving types, and thus everything must be solved using relational operations. On the other hand, there are systems that implement MO databases, typically *SECONDO* and *Hermes*. However, these systems are not aimed at building mobility DWs, therefore, implementing a mobility DW based on them is not a trivial task. Section 6 further elaborates on the discussion above.

The work presented in this paper extends and dramatically improves existing trajectory DW proposals. The implementation of *MobilityDB* gives the possibility of defining MOs as measures in a DW fact table. Integrating relational warehouse data with MO data allows realizing the notion of spatiotemporal queries defined in [4], as this paper will show through a collection of comprehensive examples. The resulting DW is called *Mobility DW*. In addition, the problem of conceptual and logical modeling of mobility DWs is also covered here. Throughout this paper, the well-known *Northwind DW* [8], extended with MO and spatial data, is used as a running example.

Note that although the work presented here is based on a centralized PostgreSQL database management system (DBMS), there are ongoing efforts to develop parallel version of this DBMS, from which *MobilityDB* (and therefore, the mobility DW studied in this paper) can benefit. Further, in a Big Data scenario, horizontal scalability is crucial. This has encouraged the development of a horizontally-scalable version of Postgres, called *Postgres-XL*, <https://www.postgres-xl.org>, which can handle mixed workloads, which include OLTP and OLAP queries, GIS queries, OLTP transactions, and so on. Last, but not least, the approach presented here can be extended to big data hadoop-based environments that are currently available, <http://spatialhadoop.cs.umn.edu>, <http://esri.github.io/gis-tools-for-hadoop>.

1.3. Paper Organization

In Section 2, the notion of temporal types is defined and explained. Section 3 describes the implementation of temporal types in *MobilityDB*. Section 4 studies the modeling of mobility DWs, also providing a background on DW design, for the non-expert readers, while Section 5 presents a comprehensive set of queries to illustrate how a mobility DW can be exploited. Section 6 discusses related work and compares such work against the present proposal, and Section 7 concludes the paper.

2. Background: Temporal Types

To make this paper self-contained, this section presents some basic concepts on temporal types. Many of the concepts and operations can be found in [1,8]. However, to develop *MobilityDB*,

new operations had been defined, and are also explained here. Also, a syntax for these temporal types is introduced here.

Temporal types are functions that map time instants to values from a domain. They are implemented by means of applying a constructor $\text{temporal}(\cdot)$. For example, a value of type $\text{temporal}(\text{integer})$ is a continuous function $f : \text{instant} \rightarrow \text{integer}$. Analogously, a temporal boolean is a function mapping instants to boolean values. In temporal databases terminology, several notions for time can be defined. The most popular ones are valid time and transaction time [9]. The former represents the time period during which a database fact is valid in the modeled reality. The latter is the time period during which a fact stored in the database is considered to be true. These are orthogonal concepts. In the remainder, valid time is considered.

Temporal types may be undefined during certain time periods. For example, in Figure 1, Alice's salary is undefined between 1 July 2017 and 1 January 2018 (e.g., because she took a six-month leave), and this is denoted ' \perp '. As a convention, closed-open intervals are used.

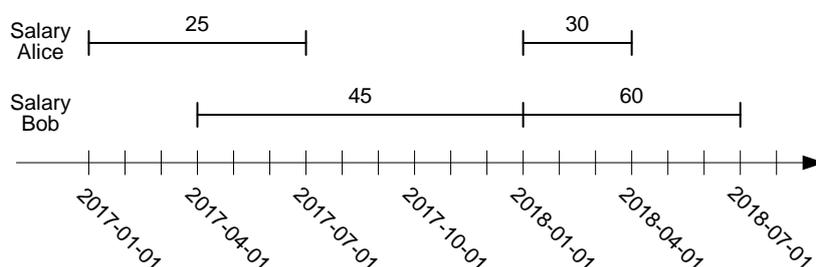


Figure 1. Representation of the evolution of salaries of two employees using the type $\text{temporal}(\text{integer})$.

2.1. Classes of Operations on Temporal Types

Temporal types are associated with different kinds of operations, described next.

Projection over domain and range. Operations of this kind are, for example, GetTime and GetValues , which return, respectively, the domain and range of a temporal type.

Interaction with domain and range. Example of operations of this kind are AtTimestamp , AtTimestampSet , AtPeriod , and AtPeriodSet , which restrict the function to a given timestamp (set) or period (set). Operations StartTimestamp and StartValue return, respectively, the first timestamp at which the function is defined and the corresponding value. The operations EndTimestamp and EndValue are analogous. The operations AtValue and AtRange restrict the temporal type to a value or to a range of values in the range of the function. The operations AtMin and AtMax restrict the function to the instants when its value is minimal or maximal, respectively.

Example 1. The functions $\text{GetTime}(\text{SalaryAlice})$ and $\text{GetValues}(\text{SalaryAlice})$ return, respectively, $\{[2017-01-01, 2017-07-01], [2018-01-01, 2018-04-01]\}$ and $\{25, 30\}$. Further, $\text{AtTimestamp}(\text{SalaryAlice}, 2017-07-15)$ returns ' \perp ', since Alice's salary is undefined at that date. The operation $\text{StartTimestamp}(\text{SalaryAlice})$ returns 2017-01-01, while $\text{AtValue}(\text{SalaryAlice}, 25)$ and $\text{AtValue}(\text{SalaryAlice}, 35)$ return, respectively, a temporal real with value $20@[2017-01-01, 2017-07-01)$ and ' \perp ', because no salary with value 35 exists.

Temporal aggregation. These kinds of operations are crucial to mobility data analysis. Three basic operations take as argument a temporal integer or real and return a real value namely: Integral , that returns the area under the curve defined by the function, Duration , which returns the duration of the temporal extent over which the function is defined, and Length , which returns the length of the curve defined by the function. From these operations, other derived operations can be defined, such as TWAvg , TWVariance , or TWStDev . The operation TWAvg computes the time-weighted average of a temporal value, taking into account the duration in which the function takes a value. TWVariance and TWStDev compute the variance and the standard deviation of a temporal type. Finally, MinValue and

MaxValue return, respectively, the minimum and maximum value taken by the function. These can be obtained by $\text{Min}(\text{GetValues}(\cdot))$ and $\text{Max}(\text{GetValues}(\cdot))$ where Min and Max are the classic operations over numeric values.

Example 2. In the example above, $\text{TAvg}(\text{SalaryAlice})$ would yield a value of 26.66, given that Alice had a salary of 25 during 181 days and a salary of 30 during 90 days.

Lifting. This class represents the generalization of the operations on nontemporal types to temporal types [10]. An operation for nontemporal types is lifted to allow any of the arguments to be replaced by a temporal type, and returns a temporal type. For example, the “less than” ($<$) operation has lifted versions (denoted by $\#<$) where one or both of its arguments can be temporal types, yielding a temporal boolean. Intuitively, the result is computed at each instant using the nonlifted operation. When two temporal values are defined on different temporal extents, the result of a lifted operation can be defined either over the intersection of both extents or the union of them. In the remainder, for the lifted operations, the first option is assumed.

Example 3. In Figure 1, the comparison $\text{SalaryAlice} < \text{SalaryBob}$ results in a temporal boolean with value $\text{true}@{\{[2017-04-01, 2017-07-01), [2018-01-01, 2018-04-01)\}}$.

Lifted aggregation operations are such that the aggregate function is computed at each time instant, and the result is defined over the union of all the extents. Figure 2 shows an example of a lifted average for the two salaries in Figure 1.

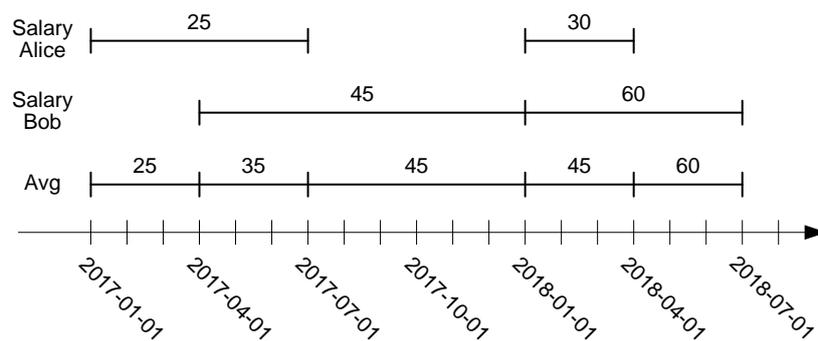


Figure 2. Example of the temporal average operation.

2.2. Temporal Types over Spatial Data

Temporal types can be also defined over spatial data types, not only over basic types like integers or reals. For example, the trajectory of a truck can be represented using a temporal(point) type. Analogously to what was explained above, this type would be a continuous function $f : \text{instant} \rightarrow \text{point}$. Some of the operations described in Section 2.1 are explained next for the spatial case. The example in Figure 3 shows two temporal points RouteT1 and RouteT2 that represent the delivery routes of two trucks T1 and T2 on a particular day. It can be seen that, for instance, truck T1 took 15 min to go from point (3, 3) to point (0, 0), while truck T2 took 15 min to go from point (4, 0) to point (1, 3). A constant speed between consecutive pairs of points is assumed.

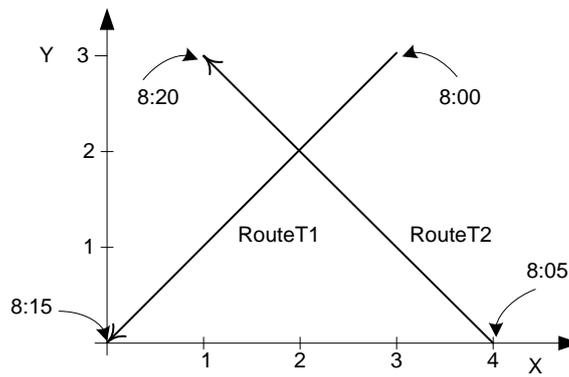


Figure 3. Trajectories of two trucks as a projection of a temporal point into the plane.

An operation called Trajectory, projects a temporal geometry into the plane. In the example, Trajectory(RouteT1) results in the leftmost line in Figure 3, without any temporal information. Note that the projection of a temporal point into the plane may consist in points and lines.

As an example of an operation of the class addressing the interaction with domain and range (as in Section 2.1) for the spatial case, the AtGeometry operation restricts the temporal point to a given geometry. For example, if Polygon denotes a polygon defined as follows Polygon((0 0, 0 2, 2 2, 2 0, 0 0)), then AtGeometry(RouteT1, Polygon) will return the value RouteT1 restricted to the period [8:05, 8:15].

Analogously to the non-spatial case, all operations over nontemporal spatial types are lifted. For example, the Distance function has lifted versions where one or both of its arguments can be temporal points and the result is a temporal real. In the example, Distance(RouteT1, RouteT2) returns a temporal real shown in Figure 4, where, for instance, the value evolves from $\sqrt{8}$ @8:05 to 2@8:10 to $\sqrt{8}$ @8:15. Note that in this case, the distance function has been approximated by a linear function.

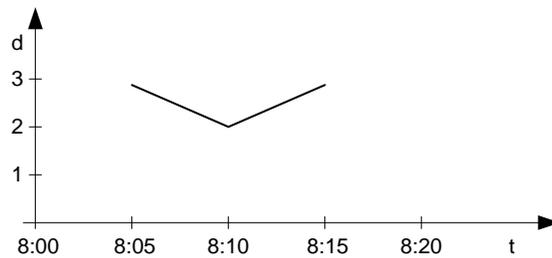


Figure 4. Distance between the trajectories of the two trucks in Figure 3 represented as a temporal(real).

Lifted topological operations return a temporal boolean. Examples of such operators are TIntersects, TDWithin, and TContains. For example, TIntersects(RouteT1, RouteT2) returns a temporal boolean with value false@[8:05, 8:15] since the two trucks were never at the same point at any instant of their route. Similarly, TDWithin(RouteT1, RouteT2, 2) returns a temporal boolean with value {false@[8:05, 8:10], true@[8:10, 8:10], false@[8:10, 8:15]} since the two trucks were only at a distance of two at 8:10. Finally, if Polygon is defined as above, then TContains(Polygon, RouteT1) will return a temporal boolean with value {false@[8:00, 8:05], true@[8:05, 8:15]}.

To conclude, several operations compute the rate of change for points. Operation Speed yields the usual concept of speed of a temporal point at any instant as a temporal real. Operation Direction returns the direction of the movement, that is, the angle between the x-axis and the tangent to the trajectory of the moving point. Finally, the Turn operation yields the change of direction at any instant.

3. Temporal Types in MobilityDB

Temporal support has been introduced in the SQL standard, and implemented (in a limited way) in commercial database systems, by means of adding temporality to tables and associating a period with each row. Nevertheless, for mobility applications this approach does not suffice. For such applications,

the temporal evolution of the attribute values is needed, along the lines of the data model proposed by Gadia and Nair [11]. This section describes the implementation in MobilityDB, of the temporal types presented in Section 2 as mentioned, a mobility database based on PostgreSQL and PostGIS. For full details, the reader is referred to the manual (<http://demo.mobilitydb.eu/mobilitydb-manual.pdf>).

3.1. Types in MobilityDB

In order to manipulate temporal types, MobilityDB uses the `timestamptz` (a shorthand for `timestamp with time zone`) type provided by PostgreSQL, and three new types: `period`, `timestampset`, and `periodset`. The `period` type is a specialized version of the `tstzrange` (short for `timestamp with time zone range`) type provided by PostgreSQL. Its functionality is similar to the one of type `tstzrange`, but with a more efficient implementation. A value of the `period` type has two bounds, the lower bound and the upper bound, which are `timestamptz` values. The bounds can be inclusive (represented by “[” and “]”), or exclusive (represented by “(” and “)”). A `period` value with equal and inclusive bounds corresponds to a `timestamptz` value. An example of a `period` value is as follows

```
SELECT period '[2017-01-01 08:00:00, 2017-01-03 09:30:00]';
```

The `timestampset` type represents a set of distinct `timestamptz` values. A `timestampset` value must contain at least one element, in which case it corresponds to a `timestamptz` value. The elements composing a `timestampset` value must be ordered. An example of a `timestampset` value is as follows

```
SELECT timestampset '{2017-01-01 08:00:00, 2017-01-03 09:30:00}';
```

Finally, the `periodset` type represents a set of disjoint `period` values. A `periodset` value must contain at least one element, in which case it corresponds to a `period` value. The elements composing a `periodset` value must be ordered. An example of a `periodset` value is as follows

```
SELECT periodset '{{2017-01-01 08:00:00, 2017-01-01 08:10:00},
[2017-01-01 08:20:00, '2017-01-01 08:40:00]}';
```

Currently, MobilityDB provides six built-in temporal types, `tbool`, `tint`, `tfloat`, `ttext`, `tgeompoint`, and `tgeogpoint`, which are, respectively, based on the `bool`, `int`, `float`, and `text` types provided by PostgreSQL, as well as the `geometry` and `geography` types provided by PostGIS (the last two types restricted to 2D and 3D points). Temporal types may be discrete or continuous. Discrete temporal types (which are based on the `boolean`, `int`, or `text` types) evolve in a stepwise manner, while continuous temporal types (which are based on the `float`, `geometry`, or `geography` types) evolve in a continuous manner. The duration of a temporal value indicates the temporal extent at which the evolution of values is recorded.

Temporal values come in four durations, namely, `instant`, `instant set`, `sequence`, and `sequence set`. A temporal `instant` value represents the value at a time instant, such as

```
SELECT tfloat '17.1@2018-01-01 08:00:00';
```

A temporal `instant set` value represents the evolution of the value at a set of time instants, where the values between these instants are unknown. For example:

```
SELECT tfloat '{17.1@2018-01-01 08:00:00, 17.5@2018-01-01 08:05:00, 18.1@2018-01-01 08:10:00}';
```

A temporal `sequence` value represents the evolution of the value during a sequence of time instants, where the values between these instants are interpolated using either a stepwise or a linear function. An example is as follows:

```
SELECT tint '(10@2018-01-01 08:00:00, 20@2018-01-01 08:05:00, 15@2018-01-01 08:10:00)';
```

As can be seen, a value of a sequence type has a lower and an upper bound that can be inclusive (represented by '[' and ']') or exclusive (represented by '(' and ')'). The value of a temporal sequence is interpreted by assuming that the period of time defined by every pair of consecutive values $v1@t1$ and $v2@t2$ is lower inclusive and upper exclusive, unless they are the first or the last instants of the sequence and in that case the bounds of the whole sequence apply. Furthermore, the value taken by the temporal sequence between two consecutive instants depends on whether the subtype is discrete or continuous. For example, the temporal sequence above represents that the value is 10 during (2018-01-01 08:00:00, 2018-01-01 08:05:00), 20 during [2018-01-01 08:05:00, 2018-01-01 08:10:00), and 15 at the end instant 2018-01-01 08:10:00. On the other hand, the following temporal sequence

```
SELECT tfloat '(10.1@2018-01-01 08:00:00, 20.2@2018-01-01 08:05:00, 15.2@2018-01-01 08:10:00)';
```

represents that the value evolves linearly from 10 to 20 during (2018-01-01 08:00:00, 2018-01-01 08:05:00) and evolves from 20 to 15 during [2018-01-01 08:05:00, 2018-01-01 08:10:00).

Finally, a temporal sequence set value represents the evolution of the value at a set of sequences, where the values between these sequences are unknown, for example:

```
SELECT tfloat '{{[17.2@2018-01-01 08:00:00, 17.5@2018-01-01 08:05:00],
               [18.2@2018-01-01 08:10:00, 18.5@2018-01-01 08:15:00]}}';
```

3.2. Using Temporal Types

The operations for temporal types defined in Section 2 can be expressed in MobilityDB as explained next. For this, the following table definition is used:

```
CREATE TABLE Employee (
    SSN CHAR(9) PRIMARY KEY,
    FirstName VARCHAR(30),
    LastName VARCHAR(30),
    BirthDate DATE,
    SalaryHist TINT );
```

Tuples can be inserted in this table as follows:

```
INSERT INTO Employee VALUES
( '123456789', 'Alice', 'Cooper', '1980-01-01',
  TINT '{{[25@2017-01-01, 25@2017-07-01], [30@2018-01-01, 30@2018-04-01]}}'),
( '345345345', 'Bob', 'Brown', '1985-07-25',
  TINT '[45@2017-04-01, 60@2018-01-01, 60@2018-04-01)');
```

The values for the SalaryHist attribute above corresponds to those in Figure 1.

Given the above table with the two tuples inserted, the query

```
SELECT GetTime(E.SalaryHist), GetValues(E.SalaryHist)
FROM Employee E
```

returns the following values

```
{[2017-01-01, 2017-07-01], [2018-01-01, 2018-04-01]} {25,30}
{[2017-04-01, 2018-07-01]} {45,60}
```

The first column of the result above is of type periodset, while the second column is of type integer[] (array of integers) provided by PostgreSQL. Similarly, the query

```
SELECT ValueAtTimestamp(E.SalaryHist, '2017-04-15'),
       ValueAtTimestamp(E.SalaryHist, '2017-07-15')
FROM Employee E
```

returns the following values

```
25  NULL
45  45
```

where the NULL value above represents the fact that the salary of the Alice is undefined on 2017-07-15. The following query

```
SELECT AtPeriod(E.SalaryHist, '[2017-04-01, 2017-11-01]')
FROM Employee E
```

returns

```
{[25@2017-04-01, 25@2017-07-01]}
{[45@2017-04-01, 45@2017-11-01]}
```

Note that here, the temporal attributes have been restricted to the period given in the query. The next query is an example of aggregation, and asks for the minimum and maximum values, together with the instants or periods when they occurred:

```
SELECT AtMin(E.SalaryHist), AtMax(E.SalaryHist)
FROM Employee E
```

The result is:

```
{[25@2017-01-01, 25@2017-07-01]}  {[30@2018-01-01, 30@2018-04-01]}
{[45@2017-04-01, 45@2017-10-01]}  {[60@2017-10-01, 60@2018-07-01]}
```

The use of lifted operations in MobilityDB is illustrated next. The following query asks for the time periods where the salary of Alice was lower than the one of Bob.

```
SELECT E1.SalaryHist #< E2.SalaryHist
FROM Employee E1, Employee E2
WHERE E1.FirstName = 'Alice' and E2.FirstName = 'Bob'
```

The query returns the temporal boolean value

```
{[t@2017-04-01, t@2017-07-01], [t@2018-01-01, t@2018-04-01]}
```

Note that when Alice's salary is undefined, no comparison is performed. As an example of lifted aggregation, the next query asks for the average salary across time.

```
SELECT AVG(E.SalaryHist)
FROM Employee E
```

returns

```
{[25@2017-01-01, 25@2017-04-01], [35@2017-04-01, 35@2017-07-01],
 [45@2017-07-01, 45@2018-04-01], [60@2018-04-01, 60@2018-07-01]}
```

Figure 2 shows this result graphically.

To conclude, the next example illustrates how temporal point types can be used. Consider the following table, which stores the routes followed by trucks to deliver products.

```
CREATE TABLE Delivery (
    TruckId CHAR(6) PRIMARY KEY,
    DeliveryDate DATE,
    Route TGEOMPOINT )
```

Next, two tuples are inserted in this table, containing information of two deliveries performed by two trucks T1 and T2 during the same day (as depicted in Figure 3):

```
INSERT INTO Delivery VALUES
('T1', '2017-01-10',
 TGEOMPOINT '[Point(3 3)@2017-01-10 08:00, Point(0 0)@2017-01-10 08:15]'),
('T2', '2017-01-10',
 TGEOMPOINT '[Point(4 0)@2017-01-10 08:05, Point(1 3)@2017-01-10 08:20]');
```

The routes represent continuous trajectories. Therefore, a constant speed between any two consecutive points is assumed, and linear interpolation for determining the position of the trucks at any instant is used. Examples of lifted spatial operations are given next.

The following query computes the distance between the two trucks at any time instant.

```
SELECT Distance(D1.Route, D2.Route)
FROM Delivery D1, Delivery D2
WHERE D1.TruckId = 'T1' AND D2.TruckId = 'T2'
```

This query returns the temporal float depicted in Figure 4 as follows:

```
[2.82842712474619@2017-01-10 08:05:00, 2@2017-01-10 08:10:00,
 2.82842712474619@2017-01-10 08:15:00]
```

Finally, the following query uses the lifted `TIntersects` topological operation for testing whether or not the two trucks intersect, as follows:

```
SELECT tintersects(D1.Route, D2.Route)
FROM Delivery D1, Delivery D2
WHERE D1.TruckId = 'T1' AND D2.TruckId = 'T2'
```

and the result will be

```
[false@2017-01-10 08:05:00, false@2017-01-10 08:15:00]
```

4. Modeling Mobility Data Warehouses

This section studies how DWs can be extended with temporal types in order to support the analysis of mobility data. The well-known Northwind case study is used in order to introduce the main concepts. First, basic concepts about DW modeling are introduced to make this paper self-contained.

4.1. A Short Introduction to Conceptual Modeling of Data Warehouses

The conventional database design process includes the creation of database schemas at the conceptual, logical, and physical levels. Typically, databases are designed at the conceptual level using some variation of the well-known entity-relationship (ER) model, since it has been acknowledged that conceptual models allow better communication between designers and users for the purpose of understanding application requirements.

Opposite to conventional databases, there is no widely accepted conceptual model for multidimensional data. As a consequence, DW design is usually at the logical level, based on star and/or snowflake schemas [3], which are less intuitive for the final user. The present paper adopts the MultiDim model [8] to represent, at the conceptual level all elements required in DW and OLAP applications, that is, dimensions, hierarchies, and facts with their associated measures. The main reason to adopt this model is that it has also been extended to support spatial data. A streamlined description of the main components of the model follows, based on Figure 5, which represents a DW for the well-known Northwind database.

using pictograms in parent-child relationships. For example, the topological constraints in dimension Geography indicate that a city is contained in its parent state and similarly for the other parent-child relationships in the hierarchy.

The fact Delivery has an identifier DeliveryID, and eight measures. The first one, Route, keeps the position of the truck at any point in time. It is a *spatiotemporal measure* of type temporal point, as indicated by the symbol 't(•)'. The other measures are derived from Route. Measure Trajectory stores the geometry of the route traversed by the truck, which is a line, without any temporal information. Measures AvgSpeed, Duration, and NbCustomers are numeric, while StartTime and EndTime are timestamps.

In the example of Figure 6, the movement track of the deliveries is represented as a temporal point. In other words, the trajectory is a measure of this fact, allowing deliveries to be aggregated along the different dimensions. Another use of trajectory aggregation identifies “similar” trajectories and merges them in a single class. Thus, queries like “give me the total number of trajectories by class,” or “List all the trajectories similar to the one followed by truck T1 on 25 November 2018” are possible.

Finally, note that the road network is not linked to any fact or dimension in the schema. This is a major difference with respect to other approaches that segment trajectories, and link trajectory segments to road segments.

The translation of the conceptual schema in Figure 6 into a snowflake schema is shown in Figure 7. The many-to-many relationship between the Delivery fact and the Customer dimension is represented in table DeliveryCustomer, which contains the keys of the tables Delivery and Customer and the attribute Sequence, which states the order in which the customer were visited. The rest of the translation is straightforward, and it is omitted for the sake of brevity.

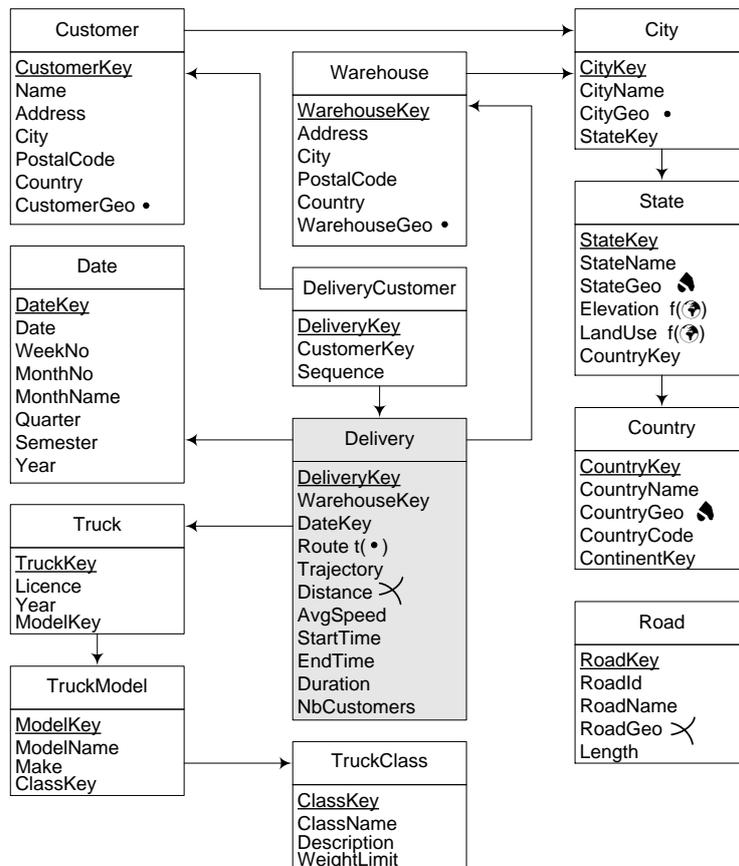


Figure 7. Logical schema of the Northwind mobility data warehouse.

5. Querying the Mobility Data Warehouse in SQL and MobilityDB

In order to express queries to the Northwind mobility DW, the temporal types and their associated operations as defined in Section 3 are used. The mobility DW depicted in Figure 7 is used as an example. Queries were targeted for Belgium, in which the administrative divisions corresponding to states are called provinces. In what follows, the functions that are prefixed by ST_ are PostGIS functions, while the functions not prefixed (e.g., length) are the temporal extensions from MobilityDB. The queries below are aimed at highlighting the advantages of defining MO data as a measure rather than the typical solution of defining static segments of trajectories as measures [5–7]. Therefore, the queries combine typical OLAP operations, like Roll-up, Slice, and Dice, with operations on MOs (in this case, the trucks that perform the deliveries. For each query, the operations involved are explained, to remark the wide variety of queries that the model and implementation allow.

Query 1 (Roll-up + slice + distance). *List the total distance traveled by trucks, per make and month.*

This query involves aggregation along the Date and Truck dimensions, slicing out all other dimensions, and the computation of the distance travelled by the MOs.

```
SELECT  M.Make, DT.Year, DT.MonthNumber, SUM(length(D.Route))
FROM    Delivery D, Truck T, TruckModel M, Date DT
WHERE   D.TruckKey = T.TruckKey AND T.ModelKey = M.ModelKey AND
        D.DateKey = DT.DateKey
GROUP BY M.Make, DT.Year, DT.MonthNumber
ORDER BY M.Make, DT.Year, DT.MonthNumber
```

Also, the query uses the function length to compute the distance traveled by each delivery, and then aggregates the distances per truck make and month in the usual way.

An analysis of this query makes it clear the advantage of the approach presented here, with respect to the approaches that segment the trajectories in order to provide a relational representation of a trajectory. The latter would require, for example, to compute the length of each segment individually (see [8], Chapter 12, for a detailed explanation), and also joining each segment with other dimension tables. Further, proposals that partition the space into a grid, precomputing aggregated measures for each cell in the grid (see Section 6), of course cannot express this query, neither any of the queries in the next examples. Also, even though MO databases like SECONDO and Hermes (also see Section 6) could of course address the MO part of the queries, integrating them into a system to perform OLAP operations is far from being trivial. Therefore, it can be seen that the mobility DW approach based on MobilityDB takes the best of both worlds, bridging the gap between them. The explanation above applies to all queries that are discussed below in this section, thus, it will not be repeated to avoid redundancy.

Query 2 (Roll-up + slice + dice + duration). *List the average duration per day of deliveries starting with a customer located in the province of Namur.*

This query requires aggregation along the Date dimension, a Roll-up operation over the Customer dimension, a Dice operation to select the province of Namur at the State level, and the computation of the duration of the time spanned by the deliveries. Finally, all dimensions but Date are sliced out.

```
SELECT  DT.Date, AVG(duration(D.Route))
FROM    Delivery D, DeliveryCustomer DC, Customer C, State S, Date DT
WHERE   D.DeliveryKey = DC.DeliveryKey AND DC.Sequence = 1 AND
        DC.CustomerKey = C.CustomerKey AND C.StateKey = S.StateKey AND
        S.Name = 'Namur' AND D.DateKey = DT.DateKey
GROUP BY DT.Date
ORDER BY DT.Date
```

The query selects the first customer visited by a delivery using the attribute Sequence, and verifies that she is located in the province of Namur. Then, it uses function duration to compute the duration of the delivery. Finally, the query computes the average of the durations per day.

Query 3 (Roll-up + slice + dice + spatial + duration). *For each month, compute the number of deliveries such that their route intersects the province of Namur for more than 20 min.*

The query performs a roll-up operation along the Date and Customer dimensions, a Dice operation to select the province of Namur at the State level, a spatial operation to compute the intersection, and the computation of the duration of the time spanned by the deliveries within the province. Finally, all dimensions but Date are sliced out.

```
SELECT  T.Year, T.MonthNumber, COUNT(*)
FROM    Delivery D, Date T, State S
WHERE   D.DateKey = T.DateKey AND S.Name = 'Namur' AND D.Route && S.Geom AND
        duration(atGeometry(D.Route, S.Geom)) >= interval '20 min'
GROUP BY T.Year, T.MonthNumber
ORDER BY T.Year, T.MonthNumber
```

This query uses the function atGeometry to restrict the route of the delivery to the geometry of the province of Namur. Then, function duration computes the time spent within the province and verifies that this duration is at least of 20 min. Remark that this duration of 20 min may not be continuous. Finally, the count of the selected deliveries is performed as usual.

The term D.Route && S.Geom is optional and it is included to enhance query performance. It verifies, using an index, that the spatio-temporal bounding box of the delivery projected over the spatial dimension intersects with the bounding box of the geometry. In this way, the atGeometry function is only applied to the deliveries satisfying the bounding box condition.

Query 4 (Roll-up + slice + dice + spatial + duration). *Same as the previous query but with the condition that the route intersects the province of Namur for more than consecutive 20 min.*

The operations involved here are similar to the ones in the previous query. The queries differ in the way in which they compute the duration. Again, this is only possible when measures are actually represented as MOs.

```
SELECT  DT.Year, DT.MonthNumber, COUNT(*)
FROM    Delivery D, Date DT, State S,
        unnest(periods(getTime(atGeometry(D.Route, S.Geom)))) P
WHERE   D.DateKey = DT.DateKey AND S.Name = 'Namur' AND
        D.Route && S.Geom AND duration(P) >= interval '20 min'
GROUP BY DT.Year, DT.MonthNumber
ORDER BY DT.Year, DT.MonthNumber
```

As in the previous query, the function atGeometry restricts the route of the delivery to the province of Namur. This results in a route that may be discontinuous in time, because the route may enter and leave the province. The query uses the function getTime to obtain the set of periods of the restricted delivery, represented as a periodSet value. The function periods converts the latter value into an array of periods, and the PostgreSQL function unnest expands this array to a set of rows, each row containing a period P. Then, it is possible to verify that the duration of one of the periods P has at least 20 min.

Query 5 (Roll-up + slice + distance + speed). *For each month, compute the total number of deliveries that traveled at least 50 km at a speed higher than 100 km/h.*

The operations involved in this query are: a Roll-up along the Date dimension, and the computation of the distance and speed functions on the MO data. Finally, slicing is performed to drop all dimensions but Date.

```

SELECT  DT.Year, DT.MonthNumber, COUNT(*)
FROM    Delivery D, Date DT
WHERE   D.DateKey = DT.DateKey AND length(atPeriodSet(D.Route,
getTIme(atRange(speed(Route) * 3.6, '[100,infinity]')))) / 1000 > 50
GROUP BY DT.Year, DT.MonthNumber
ORDER BY DT.Year, DT.MonthNumber

```

The query uses the function speed to obtain a temporal float representing the speed of the route at each instant, expressed in units per second, depending on the spatial reference system, which in our case is meters per second. This temporal value is then transformed into kilometers per hour by multiplying it by 3.6. The function atRange restricts the speed to the float range [100, infinity]. The getTIme function computes the set of periods during which the delivery travels at more than 100 km/h. The function atPeriodSet restricts the delivery to these periods and the length function computes the distance traveled by the restricted deliveries, expressed in meters, which is then converted to kilometers and compared against the value 50.

Query 6 (Temporal aggregation). *For each speed range of 20 km/h, give the total distance traveled by all trucks within that range.*

This is a typical temporal aggregation query, as explained below. That is, operations over MOs are performed over the measure, without involving the dimensions. Only the fact table is used here.

```

WITH Ranges AS (
    SELECT I AS RangeID, floatrange(((I-1)*20), I*20) AS Range
    FROM generate_series(1,10) I )
SELECT  R.Range, SUM(length(atPeriodSet(D.Route,
getTIme(atRange(speed(D.Route) * 3.6, R.Range)))) / 1000)
FROM    Delivery D, Ranges R
WHERE   atRange(speed(D.Route) * 3.6, R.Range) IS NOT NULL
GROUP BY R.RangeID, R.Range
ORDER BY R.RangeID

```

This idea of this query is to allow having an overall view of the speed behaviour of the entire fleet of delivery trucks. The temporary table Ranges stores the ranges [0, 20), [20, 40), ... [180, 200). In the main query, the speed of the route, obtained in meters per second, is transformed into kilometers per hour. Then, the function atRange restricts the route to the portions having a given speed range. The time periods comprising the restricted route are obtained with the getTIme function. The overall route is restricted then to the obtained time periods with the function atPeriodSet, the distance traveled at the given speed range is obtained with the length function, and all the distances for all trucks at the given time range are obtained with the SUM aggregation function.

Query 7 (Slice + spatial). *Compute the number of deliveries that traversed at least two states.*

The OLAP operation here is a Slice, to drop all dimensions. The rest of the query involves spatial operations to perform intersections between the MOs' trajectories and the State dimension level. Note that, in fact, the intersection is performing a "climbing" along the geographic dimension, skipping the intermediate levels in the hierarchy.

```

SELECT  COUNT(*)
FROM    Delivery D, State S1, State S2
WHERE   S1.StakeyKey <> S2.StateKey AND
        ST_Intersects(trajjectory(D.Route), S1.Geom) AND
        ST_Intersects(trajjectory(D.Route), S2.Geom)

```

This query projects the route of the delivery to the spatial dimension using the function trajectory, which results in a geometry. The query then tests that the trajectory of the route intersects two states using the PostGIS function ST_Intersects.

Query 8 (Slice + distance + duration). *List the pairs of deliveries that traveled at less than one kilometer from each other during more than half an hour. Again, the OLAP operation here is a Slice, to drop all dimensions. Only the Delivery fact is kept. Then, computations on the temporal types are performed.*

```
SELECT D1.DeliveryKey, D2.DeliveryKey
FROM Delivery D1, Delivery D2
WHERE D1.DeliveryKey <> D2.DeliveryKey AND duration(getTime(
    atValue(tdwithin(D1.Route, D2.Route, 1000), TRUE))) > '20 min'
```

The function tdwithin is the temporal version of the PostGIS function ST_DWithin. It returns a temporal boolean which is true during the periods when the two routes are within the specified distance from each other. The function atValue restricts the temporal boolean to the periods when its value is true, and the function getTime obtains these periods. Finally, the query uses the duration function to obtain the corresponding interval and verifies that it is greater than 20 min.

The next example query involves the LandUse continuous field.

Query 9 (Roll-up + slice + spatial (raster) + distance + duration). *Compute the average duration of the deliveries that started in a residential area and ended in an industrial area on 1 February 2017.*

The OLAP operations are a slice, to drop all dimensions but Date and Customer, and a roll-up over the Customer (only up to the bottom level) and Date dimensions. Spatial operations over raster data are also performed. Finally, also computations on the temporal types are performed.

```
SELECT AVG(duration(D.Route))
FROM Delivery D, Date T, DeliveryCustomer DC1, DeliveryCustomer DC2,
    Customer C1, Customer C2, City Y1, City Y2, State S1, State S2
WHERE D.DateKey = T.DateKey AND D.Date = '2017-02-01' AND
    D.DeliveryKey = DC1.DeliveryKey AND DC1.Sequence = 1 AND
    D.DeliveryKey = DC2.DeliveryKey AND DC2.Sequence = D.NbCustomers AND
    DC1.CustomerKey = C1.CustomerKey AND
    DC2.CustomerKey = C2.CustomerKey AND
    C1.CityKey = Y1.CityKey AND C2.CityKey = Y2.CityKey AND
    Y1.StateKey = S1.StateKey AND Y2.StateKey = S2.StateKey AND
    ST_Intersects(C1.CustomerGeo, atValue(S1.LandUse, 'Residential')) AND
    ST_Intersects(C2.CustomerGeo, atValue(S2.LandUse, 'Industrial'))
```

The query starts by selecting the deliveries of the given date. It also selects, using the bridge table DeliveryCustomer, the first and last customers served by the delivery. Then, the query obtains the state of these customers with the subsequent joins. Further, the function AtValue restricts the land use fields of the corresponding states to the values of type residential or industrial. Finally, the function ST_Intersects ensures that the start and end locations of the delivery are included in the restricted fields. Notice that, in PostGIS, the ST_Intersects predicate can compute not only if two geometries intersect, but also if a geometry and a raster intersect.

Note that the query above does not involve temporal data since it neither mentions a temporal geometry such as measure Route, nor a temporal field such as Temperature. The next query involves both temporal attributes, and combines a field and a trajectory.

Query 10 (Slice + spatial (raster) + distance). *List the deliveries that have traveled along more than 50 km of roads at more than 1000 m of altitude.*

A slice is performed to drop all dimensions. Spatial operations over raster data (in this case, representing elevation) are performed. Finally, computations on the temporal types are also performed.

```
SELECT D.DeliveryNumber
FROM Delivery D
WHERE ( SELECT SUM(ST_Length(ST_Intersection(Trajectory(D.Route),
      AtRange(S.Elevation, '[1000,infinity]'))).geom))
      FROM State S
      WHERE ST_Intersects(Trajectory(D.Route), S.StateGeo) ) > 50,000
```

For each delivery, using the function `ST_Intersects`, the inner query verifies that the route of the delivery and the geometry of the state intersect. Then, the elevation field of the state is restricted to the range of values higher than 1000 m with function `AtRange`. The function `ST_Intersection` computes the intersection of the trajectory of the route and the restricted field, and the length of this route is computed with the function `ST_Length`. The `SUM` aggregate function is then used to calculate the sum of the lengths of all the obtained routes for all states, and finally the outer query verifies that this sum is greater than 50 km.

Discussion on Performance

Since the paper is oriented to show the features of the mobility data warehousing approach, it is focused on highlighting the expressiveness that MO data adds to the classic and spatial data warehousing approaches. Therefore, a thorough evaluation of query performance is outside the scope of this work. Nevertheless, this section reports a preliminary evaluation, developed as follows. The mobility DW depicted in Figure 7 was implemented over a PostgreSQL relational database. Delivery data (the MO data in attribute `Route` of the fact table `Delivery`) had been produced using the data generator of the BerlinMOD benchmark for moving objects <http://dna.fernuni-hagen.de/secondo/BerlinMOD/BerlinMOD.html>. However, since the BerlinMOD benchmark targets the city of Berlin, the geographic hierarchy has been dropped, and the references to the State dimension had been replaced by a Borough dimension, both in the schema and in the queries. The evaluation was performed with 520 delivery tuples. The minimum, maximum, and average length of the deliveries are 96, 4632, and 1870, respectively, therefore, this would be approximately equivalent in size to a “normalized” fact table of one million records. The rest of the data are based on the Northwind database. Thus, the table `Customer` contains 1430 tuples, and there are seven warehouses and 100 trucks. Queries 1–8 were run (Queries 9 and 10 were not run, since they include raster data just as an example), changing, as mentioned, the references to the State with references to the Borough. For example, Query 3 now reads:

```
SELECT T.Year, T.MonthNumber, COUNT(*)
FROM Delivery D, Date T, Borough B
WHERE D.DateKey = T.DateKey AND S.Name = 'Mitte' AND D.Route && B.Geom AND
      duration(atGeometry(D.Route, B.Geom)) >= interval '20 min'
GROUP BY T.Year, T.MonthNumber
ORDER BY T.Year, T.MonthNumber
```

Analogously, Query 4 reads, for these experiments:

```
SELECT DT.Year, DT.MonthNumber, COUNT(*)
FROM Delivery D, Date DT, Borough B,
      unnest(periods(getTime(atGeometry(D.Route, B.Geom)))) P
WHERE D.DateKey = DT.DateKey AND B.Name = 'Mitte' AND
      D.Route && B.Geom AND duration(P) >= interval '20 min'
GROUP BY DT.Year, DT.MonthNumber
ORDER BY DT.Year, DT.MonthNumber
```

Table 1 shows the results obtained in these tests. Queries were run several times to eliminate the influence of cold caching (that is, hot cache is assumed). Further, once this influence was eliminated, execution times were the same for each run, therefore just the average value was reported in the table, since the differences between runs were negligible. It can be seen that the queries that took longer were the ones that included spatial operations.

Table 1. Running queries.

Query	Execution Time (s)
Q1	2
Q2	0.35
Q3	180
Q4	180
Q5	12
Q6	60
Q7	60
Q8	9

Of course, these results are not pretended to be conclusive, but suggest that the addition of MOs as fact measures in a warehouse adds query expressiveness. Also, existing proposals segment trajectories (see Section 6) require a “normalization” of the trajectory representation, which is costly due to the number of joins that queries like the ones presented here would require. This is not needed when trajectories are represented as MOs. In addition, note that the DW is implemented using standard object-relational technologies (in this case, PostgreSQL and PostGIS). Therefore, the effect of the inclusion of MOs can also be inferred from an evaluation of the implementation of the temporal types. This is available in the MobilityDB demo site, at URL <http://demo.mobilitydb.eu/>, where the system runs over the BerlinMOD benchmark data (the queries in such demo do not include warehouse data, they are purely MO queries). It can be observed on the demo site that typical MO queries run very fast over the benchmark, although this is not reported in this paper. Developing a benchmark to perform a comprehensive set of experiments is outside the scope of this paper, and is planned as future work (Section 7).

6. Related Work

Trajectory DWs are strongly related to spatial databases and DWs. The main features of PostGIS, the OGC-based extension to PostgreSQL can be found in [12]. Also, the spatial extension of the MultiDim model presented here was based on the spatial data types of MADS [13], a spatiotemporal conceptual model. The notion of spatial OLAP (SOLAP) was introduced in [14], and it is reviewed in [15]. Other relevant work on SOLAP can be found in [16–18].

In order to support spatio-temporal data, a data model and associated query language for MO data were needed. This is achieved in SECONDO <http://dna.fernuni-hagen.de/secondo/> [19], a MO database system developed at the FernUniversität in Hagen, based on the model of Güting et al. [1]. Along similar lines, Hermes is a MOD introduced by Pelekis et al. [20]. The Hermes system is described in [21]. In spite of their ability to handle MO data, neither SECONDO, nor Hermes, are oriented toward addressing the problem of mobility DWs. Among other reasons, integrating both prototypes into existing relational databases is not straightforward. Hermes, for example, extends Oracle through a so-called cartridge with the MO types. However, to build an application on top of the database the application developer must write a source program (for example, in Java), and embed PL/SQL scripts that invoke object constructors and methods from Hermes. SECONDO is as packed system, therefore, integration with existing databases is even more complicated. On the contrary, being coded in the “C” programming language (like PostgreSQL), MobilityDB seamlessly extends the PostGIS library with temporal data types, not requiring any additional software architecture. Thus, building a mobility

DW with MobilityDB turns out to be a natural extension of a spatial relational DW. On the downside, MobilityDB at this time only supports moving points, while *SECONDO* and *Hermes* provide (although limited) support to other MOs. However, for traffic analysis, moving points are the most used kind of MOs. The data type system of the temporal types presented in this paper follows the approach of [1]. Also, an SQL extension for spatiotemporal data is proposed in [22].

The work by Orlando et al. [6] introduces the concept of trajectory DWs, aimed at providing the infrastructure needed to deliver advanced reporting capabilities and facilitating the use of mining algorithms on aggregate data. This work is based on the *Hermes* system. A relevant feature of this proposal is the treatment given to the extraction, transformation and loading (ETL) process, which transforms the raw location data and loads it to the trajectory DW. However, regarding mobility analysis, this proposal does not address MO data. Measures in this trajectory DW are related with the number of MO present in a cell defined by spatiotemporal coordinates. Therefore, queries that can be addressed using this DW are of the form “compute the number of cars between latitudes l_1 and l_2 , and longitudes lo_1 and lo_2 , on 3 November 2018”. However, actual mobility analysis queries like the ones presented in Section 5 cannot be addressed. Also, the authors themselves state in their proposal that MO data analysis remains outside the TDW.

Along similar lines, Wagner et al. [7] proposed the *Mob-Warehouse*, a conceptual to support mobility analysis using a TDW. The authors propose to enrich trajectory data with semantic information about the domain. The paper is based on the previously defined notion of semantic trajectory [23]. Unlike the model presented in [6], where the measures are pre-aggregated (e.g., number of cars in a cell), in this model the measures are at the finest detail level. No implementation details are given, and MO operations are not addressed. Instead, the proposal is aimed at analyzing semantic trajectories, as the example queries provided suggest.

Based on [7,23], and the work on semantic trajectories, another model for TDWs is proposed in [5], accounting for semantic information. In this case, again, MO data are only partially considered. MO trajectories are partitioned into segments, to which semantic information is associated. No implementation is described, and the authors report than experiments are being carried out, suggesting that the segments in the fact table are not that efficient, which is certainly expected given that many joins are required to reconstruct a trajectory from such segments.

Note that the proposal presented in the present paper does not exclude semantic information. On the contrary, semantic information about MOs can be naturally included in the mobility DW proposed here, and, in fact, this is what dimensions are defined for. Also, all of the features of the proposals above are supported by the model described in this paper (e.g., segmentation, pre-aggregation, semantic information).

An overall perspective of the state of the art in mobility data management can be found in [2,24–26]. A survey on spatiotemporal data warehousing, OLAP, and mining is given in [27]. A discussion about the meaning of spatiotemporal data warehousing is given in [4]. Also, mobility DWs are discussed in [28–30]. Analysis tools for mobility DWs can be found in [31]. Finally, a survey on spatiotemporal aggregation is given in [32], while a survey on trajectory aggregation is provided in [33].

7. Conclusions

This paper studied how DWs can be extended with spatial and mobility data. For this, temporal types were defined to capture the variation of a value across time. The paper discussed how these data types, representing MOs, were implemented as an extension to PostGIS. This extension allows to define a conceptual model for a trajectory DW, which includes MO data as measures that can be analyzed with respect to contextual dimensions. Therefore, the contextual model can actually be implemented with MO data, not only segmented trajectories or pre-aggregated measures. This is a key difference with previous proposals, and would not be possible without a data type system that seamlessly integrates with the (spatial) database supporting the warehouse. A concrete case study that extends the *Northwind* DW with mobility data was presented, illustrating the use of the mobility

DW with a comprehensive set of queries. Future work includes the development of more kinds of temporal types supporting not only moving points, but also moving lines and moving regions and, as mentioned in Section 5, the development of an appropriate benchmark to allow a fair evaluation of mobility DWs. Another research direction will address the extension of the work presented in this paper, to big data environments, along the lines mentioned in Section 1.

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